**BIG M  PROBLEMS**

**1*.*Use penalty (or Big  'M') method to**

**Minimize  z  = 4xi + 3x2**

 subject to the constraints :

                            2x1+ x2 ≥ 10,   -3x1, +  2x2  ≤ 6

                            x1 + x2 ≥ 6,   x1 ≥ 0 and x2  ≥ 0.

 **Solution*.*** Introducing surplus (negative slack) variables x3  ≥ 0,  x5  ≥ 0 and slack variable x4  ≥ 0 in the constraint inequations, the problem becomes

 Maximize  z\*  =  - 4x1- 3x2+ 0.x3+ 0.x4 + 0.x5

subject to the constraints :

                                    2x1 + x2 - x3 = 10,  - 3x1 + 2x2 + x4 =  6

                                      xl + x2– x5 =  6,  xj ≥ 0 Q(j = 1,2, 3, 4, 5)

 Clearly, we do not have a ready basic feasible solution. The surplus variables carry negative coefficients (-1). We introduce two new variables A1 ≥ 0 and A2 ≥ 0 in the first and third equations respectively. These extraneous variables, commonly termed as artificial variables, play the same role as that of slack variables in providing a starting basic feasible solution.

 We assign a very high penalty cost (say -*M, M* ≥ 0) to these variables in the objective function so that they may be driven to zero while reaching optimality.

 Now the following initial basic feasible solution is available :

                                         Al = 10, x4 = 6   and A2 =  6

 with **B** = (**a6 , a4, a7**) as the basis matrix. The cost matrix corresponding to basic feasible solution is **cB** = ( -*M, 0, -M* )

 Now, corresponding to the basic variables A1, x4 and A2. the matrix **Y = B-1A** and the net evaluations zj - cj(j = 1, 2, .... 7) are computed. The initial basic feasible solution is displayed in the following simplex table :



We observe that the most negative zj - cj is  4 - 3M (= zl– c1). The corresponding column vector y1, therefore, enters the basis. Further, since min.  = 5;  the element y11 (=2) becomes the leading element for the first iteration

**First Iteration:**   Introduce y2 and drop y7.

 In the above table, we omitted all entries of column vector **y6**, because the artificial variables Al has left the basis and we would not like it to re-enter in any subsequent iterations.

Now since the most negative (zj—cj) is  z2-c2; the non-basic vector **y2** enters the basis. Further, since min   is 2 which occurs for the  element y32 ( = 1/2), the corresponding basis vector **y7** leaves the basis and the element  y32 becomes the leading element for the next iteration.

 **Final Iteration:** Optimum Solution,

 It is clear from the table that all zj - cj are positive. Therefore an optimum basic feasible solution has been attained which is given by

        x1= 4, x2 = 2,  maximum  z = 22.

**2. Maximize z = 3x1, + 2x2   subject to the constraints :**

            **2x1 + x2≤ 2,  3x1 + 4x2  ≥  12, x1, x2  ≥ 0.**

**Solution**:

Introducing slack variable x3 ≥ 0,  surplus variable x5 ≥ 0 and an artificial variable *A1* ≥ 0, the reformulated L.P.P. can be written as :

Maximize z  = 3x1 + 2x2 + 0.x3 + 0.x4 – MA1

subject to the constraints :

2x1 + x2+ x3 = 2,

3x1 + 4x2- x4 + A1= 12

x1, x2, x3, x4 ≥ 0 and A1≥ 0.

An obvious   starting basic feasible solution is :

x3 = 2   and   A1 = 12.

The iterative simplex tables are :

**Initial Iteration*:*** Introduce y2 and drop y3.

**Final Iteration*.*** No solution.

 Here the coefficient of *M*in each zj - cjis non-negative and an artificial vector appears in the basis, not at the zero level. Thus the given L.P.P. does not possess any feasible solution.











